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# A High Statistics Lattice Calculation of $f_B^{static}$ at $\beta = 6.2$ using the Clover Action

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## Abstract

We present a calculation of  $f_B$  in the static limit, obtained by numerical simulation of quenched QCD, at  $\beta = 6.2$  on a  $18^3 \times 64$  lattice, using the SW-Clover quark action. The decay constant has been extracted by studying heavy(static)-light correlation functions of different smeared operators, on a sample of 220 gauge field configurations. We have obtained  $f_B^{static} = (290 \pm 15 \pm 45)$  MeV, where the first error comes from the uncertainty in the determination of the matrix element and the second comes from the uncertainty in the lattice spacing. We also obtain  $M_{B_s} - M_{B_d} = (70 \pm 10)$  MeV and  $f_{B_s}^{stat} / f_{B_d}^{stat} = 1.11(3)$ . A comparison of our results with other calculations of the same quantity is made.

In the last few years, several groups[1]-[9] have computed on the lattice the pseudoscalar decay constant in the static limit,  $f_B^{stat}$ , using the method originally proposed by E. Eichten[10]. One can combine the information obtained from the calculation of the decay constant in a range of masses near the charm mass with  $f_B^{stat}$ . In this way it is possible to reduce the uncertainty in the prediction of  $f_B$ , which, on current lattices, can only be obtained by interpolation in the heavy quark mass[6, 7, 11]. To improve the accuracy on  $f_B$  it is then important to reduce the uncertainties in the static case. These uncertainties are still rather large, as it can be seen from the large spread of values reported in the literature[1]-[9]. They are mostly due to lattice artefacts, to the calibration of the lattice spacing and to the renormalization of the lattice axial current.

To reduce lattice artefacts, two groups[7, 8] have found it convenient to use the SW-Clover improved action[12]. In a previous study we have calculated  $f_B^{stat}$  at  $\beta = 6.0$ , using cubic smeared sources[8]. In this letter we extend the study to a smaller lattice spacing,  $a$ , corresponding to  $\beta = 6.2$  (again using the SW-Clover action). In the future, a further reduction of discretization errors will eventually be obtained by comparing results at different values of  $a$  and extrapolating to the continuum. This demands a very high accuracy in the

determination of  $f_B^{stat}$  at any fixed value of the lattice spacing. Our work is a step in this direction.

In ref.[8], we found that, by using a cubic smearing source in the Coulomb gauge, there is an optimal smearing size for which it is easiest to isolate the lightest pseudoscalar state, corresponding to the B-meson in the static limit. We confirm this result and find that, using a double smeared source[1, 8] at  $\beta = 6.2$ , the optimal size is  $L_s = 11$  ( $L_s = 13$  is the optimal size for single smeared sources).  $L_s = 11$  and double smeared sources have been used to obtain the results reported in the abstract and below<sup>1</sup>. We will also compare different smearing sizes ( $7 \leq L_s \leq 13$ ).

The decay constant in the static limit can be derived from the matrix element of the static-light quark lattice axial current  $Z^L$ <sup>2</sup>:

$$f_P^{stat} \sqrt{M_P} = \sqrt{2} Z_A^{stat}(M_P a, \alpha_s) Z^L a^{-3/2} \quad (1)$$

where  $f_P^{stat}$  is the decay constant for a meson of mass  $M_P$ . The renormalization constant  $Z_A^{stat}(M_P a, \alpha_s)$  includes the anomalous dimension of the axial current in the effective theory and the  $O(\alpha_s)$  correction necessary to relate the lattice operator in the effective theory to the continuum operator in the full theory. The estimate of  $Z_A^{stat}$  is subject to a large uncertainty[13]-[17]. To evaluate it, we have used the “boosted”, tadpole improved perturbation theory proposed in ref.[18, 19]. The uncertainty in the determination of  $Z_A^{stat}$  is given by the uncertainty on the value of the boosted coupling  $g_V^2$  to be used and by higher order corrections. We have used two different definitions of  $g_V^2$ , i.e.  $g_V^2 = (8K_c)^4 6/\beta$  and  $g_V^2 = 6/\beta / <1/3 Tr U_p >[6, 18, 19]$ . With the SW-Clover action, unlike the Wilson case, the two determinations of  $g_V^2$  almost coincide and we get  $Z_A^{stat} = 0.805 - 0.812$ [15, 16]. Thus we have chosen  $Z_A^{stat} = 0.81$ . Using  $a^{-1} = (3.0 \pm 0.3)$  GeV, as suggested by the calibration of the scale from  $M_\rho$  and  $f_\pi$  from this work and ref.[28], together with  $Z^L = 0.111(6)$ , we get  $f_P^{stat} \sqrt{M_P} = (0.66 \pm 0.04 \pm 0.10)$  GeV<sup>3/2</sup> corresponding to:

$$f_B^{stat} = (290 \pm 15 \pm 45) \text{ MeV}, \quad (2)$$

where the first error comes from the uncertainty in  $Z^L$  and the second comes from the uncertainty in the scale. When the scale is taken from the string tension,  $a^{-1} \sim 2.7$  GeV, one gets a lower value, i.e.  $f_B^{stat} = (245 \pm 15)$  MeV. The calibration of the scale from the string tension is more convenient when studying the scaling properties of  $f_B^{stat}$  with  $a$ , since the error on this quantity is of  $O(a^2)$ . It is however difficult to express the lattice string tension in terms of a well defined experimental quantity, in order to obtain the absolute scale. If we use the naive perturbative value of the renormalization constant, ie.  $Z_A^{stat} = 0.88$ , and use  $a^{-1} = (3.0 \pm 0.3)$  GeV, we obtain instead  $f_B^{stat} = (315 \pm 15 \pm 45)$  MeV.

Most of the methods and techniques used in the present study have been developed and explained in the literature on the same subject[1]-[9]. Details can be found for example in a previous publication by our collaboration[8]. In the following we explain only those aspects which are new with respect to ref.[8]. The main differences are the following:

1) smearing in the origin;

2) calculation of the matrix elements of the local axial current,  $Z_L$ , by fitting the smeared-local correlation function;

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<sup>1</sup>At  $\beta = 6.0$  the optimal smearing size was found to be  $L_s = 5 - 7$ . For a more detailed discussion see ref.[8].

<sup>2</sup>The precise definition of  $Z^L$  will be given in eq.(8).

3) a method to extract  $Z_L$  from ratios of correlation functions and without any fitting procedure.

In this study, we have used local-smeared correlation functions with the smeared source in the origin. This strongly reduces the statistical noise in the correlators[20]. In our previous work at  $\beta = 6.0$  we used instead the smeared-local correlations with the smeared source in the sink. To improve the results of ref.[8], we are planning to repeat the calculation at  $\beta = 6.0$  with the smeared source in the origin. Points 2) and 3) will be discussed below.

We have generated 220 configurations in the quenched approximation, at  $\beta = 6.2$  on a  $18^3 \times 64$  lattice. The configurations were obtained from several independent runs, each with a thermalization of 3000 Montecarlo sweeps (5 hits), starting from a cold configuration. Independent configurations were gathered every 800 sweeps. We have then transformed each configuration to the Coulomb gauge, with the same accuracy as in ref.[8]. We have used the  $O(a)$  improved SW-Clover action[12]. The operators appearing in the correlation functions are “improved” by modifying the light quark propagators as explained in sec.9 of ref.[4]. The statistical errors quoted in this letter have been estimated using the jackknife method, by decimating 10 configurations at a time. In ref.[8] we verified that this gives a reliable estimate of the errors. Throughout this letter we have computed light quark correlation functions using a “thinning” trick. This method consists of the following: one only considers one point out of three in each direction when summing the correlation functions over space, at fixed time. This allows a saving of a factor of 27 in the memory required for the quark propagators. Thinning was a necessity imposed by memory limitations of the APE-tubo[21] used for the present simulation. We have checked that the effect of thinning on 2-point correlation functions is totally negligible[22].

We have inverted the light quark propagators for three values of the bare quark masses,  $K = 0.14144, 0.14190, 0.14244$ . For comparison, the first  $K$  value has been chosen equal to one of those used by the UKQCD collaboration[7], at the same value of  $\beta$ . Our lightest quark mass is slightly larger than in ref.[7] since, for lack of memory, we have to work on a smaller space volume ( the UKQCD Collaboration is working at  $\beta = 6.2$  on a  $24^3 \times 48$  lattice).

We have measured the pseudoscalar ( $M_5$ ) and vector ( $M_V$ ) meson masses and the matrix elements  $\sqrt{Z_5} = \langle 0 | \bar{\psi} \gamma_5 \psi | P \rangle$  and  $\sqrt{Z_V} = \langle 0 | \bar{\psi} \gamma_k \psi | V \rangle$  by fitting the correlation functions of the local pseudoscalar density and local vector current with components  $k = 1, 2, 3$ , in the time interval  $15 \leq t \leq 28$ . By studying several time intervals, we have checked that the contamination due to higher mass excitations induces a negligible systematic error on the masses and the matrix elements of the local operators, in the range of quark masses explored in this study. The results at each  $K$  and  $K_c$  ( the critical value of the hopping parameter) are reported in table 1. From a linear fit in  $1/K$  to  $M_5^2$  we obtain  $K_c = 0.14315(4)$ . Using  $M_K = 498$  MeV and the scale from the rho mass we obtain  $K_s = 0.1422(2)$ , where  $K_s$  is the Wilson parameter corresponding to the strange quark.

The physical size of the lattice, obtained from the vector meson mass extrapolated linearly in  $M_5^2$  to the chiral limit, is  $a^{-1} = (3.2 \pm 0.3)$  GeV. The pseudoscalar ( $f_P$ ) and vector ( $1/f_V$ ) decay constants have been extracted using standard methods, see for example ref.[8], from the local vector and axial vector currents. In table 1 we report the lattice values of  $f_P/Z_A^{Ren}$ , the ratio  $f_P/Z_A^{Ren} M_V$  and  $1/Z_V^{Ren} f_V$  as a function of the light quark mass.  $Z_{V,A}^{Ren}$  are the renormalization constants of the vector and axial vector lattice currents[23]-[27] which, for the SW-Clover action, have been computed non-perturbatively at  $\beta = 6.2$ ,

K	0.14144	0.14190	0.14244	$K_c$
$M_5$	0.292(3)	0.247(3)	0.187(4)	—
$M_V$	0.375(6)	0.338(8)	0.291(18)	0.238(21)
$Z_5 \times 10^3$	8.55(44)	7.43(46)	6.65(60)	—
$Z_V \times 10^3$	1.80(18)	1.28(18)	0.76(27)	—
$f_P/Z_A^{Ren}$	0.063(2)	0.059(2)	0.052(3)	0.045(4)
$f_P/Z_A^{Ren} M_V$	0.169(5)	0.173(7)	0.180(15)	0.186(20)
$1/Z_V^{Ren} f_V$	0.302(7)	0.313(10)	0.326(22)	0.341(27)

Table 1: Pseudoscalar and vector meson masses and matrix elements at different values of  $K$ . The corresponding values extrapolated linearly to  $K_c$  are also given.

$Z_V^{Ren} = 0.831(2)$  and  $Z_A^{Ren} = 1.04(1)[24]$ . We have extrapolated  $f_P/Z_A^{Ren}$ ,  $f_P/Z_A^{Ren} M_V$  and  $1/Z_V^{Ren} f_V$  linearly in  $M_5^2$  to the chiral limit, obtaining the numbers in table 1. Using the non-perturbative determinations of  $Z_V^{Ren}$  and  $Z_A^{Ren}$  given above, and  $M_\rho = 770$  MeV we get

$$f_\pi = (149 \pm 16) \text{ MeV} , \quad \frac{1}{f_\rho} = 0.28 \pm 0.02 \quad (3)$$

We also obtain  $f_K/f_\pi - 1 = 0.11 \pm 0.04$  and  $1/f_\phi = 0.27 \pm 0.02$ . We give for comparison the experimental values:  $f_\pi = 132$  MeV,  $1/f_\rho = 0.28$ ,  $f_K/f_\pi - 1 = 0.22$  and  $1/f_\phi = 0.23$ . The slope of the pseudoscalar decay constant as a function of the quark mass is lower than what we found in previous calculations at  $\beta = 6.0$  and for this reason we get a rather low value for  $f_K/f_\pi - 1$ . We believe however that this effect will disappear with better statistics. We can also use  $f_\pi$  to fix the scale. By using the value of  $Z_A^{Ren}$  quoted above we obtain  $a^{-1} = (2.8 \pm 0.3)$  GeV to be compared to the value obtained from  $M_\rho$  of  $a^{-1} = (3.2 \pm 0.3)$  GeV<sup>3</sup>. At  $\beta = 6.0$ , we found that the scale determined from  $f_\pi$  and  $M_\rho$  were in very good agreement. In the present case there is a difference of about 400 MeV that we cannot resolve, given the large statistical errors. We also obtain  $M_K^* = (893 \pm 7)$  MeV and  $M_\phi = (1020 \pm 13)$  MeV which compare very well with the experimental values of 892 MeV and 1019 MeV respectively.

The static B meson decay constant,  $f_B^{stat}$  has been obtained from the correlation functions of three different currents:

$$A_\mu^L(x) = \bar{Q}(x) \gamma_\mu \gamma_5 q(x) \quad (4)$$

$$A_\mu^S(x) = \sum_i \bar{Q}(x_i) \gamma_\mu \gamma_5 q(x) \quad (5)$$

$$A_\mu^D(x) = \sum_{i,j} \bar{Q}(x_i) \gamma_\mu \gamma_5 q(x_j) \quad (6)$$

where the sum over  $i$  ( $j$ ) is extended over a spatial cube of size  $L_s$ , centered at  $x$  and the heavy quark  $Q$  is taken from the static limit. We have computed the correlation functions of the above currents,  $C^{LL}$ ,  $C^{LS}$ ,  $C^{LD}$ ,  $C^{SS}$  and  $C^{DD}$ , for  $L_s = 7, 9, 11, 13$ .  $C^{LD}$  is defined as:

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<sup>3</sup>From the axial current-axial current two point function one can also get a lattice estimate of  $f_\pi$ . In this case, using the same  $Z_A^{Ren}$  and the experimental value of  $f_\pi$  we also get  $a^{-1} = (2.8 \pm 0.3)$  GeV.

$$C^{LD}(t) = \sum_{\vec{x}} \langle 0 | A_0^L(\vec{x}, t) A_0^D(\vec{0}, 0)^\dagger | 0 \rangle \quad (7)$$

and similarly for the others. At large time distances the correlation functions behave as the propagator of a single state:

$$C^{LD}(t) = Z^L Z^D e^{-\Delta E t} \quad (8)$$

The constants  $Z^L$ ,  $Z^S$  and  $Z^D$  are related to the matrix elements of the different axial currents and  $\Delta E$  is the binding energy of the lightest pseudoscalar state.

A standard method to extract  $Z^L$  is the following:

1) At large time distances the correlation functions are dominated by the lightest propagating state. Let us call  $t_i$  the time at which we start observing a plateau both for the effective binding energy:

$$\Delta E_{DD}^{eff} = \log[C^{DD}(t)/C^{DD}(t+1)] \quad (9)$$

and for the ratio:

$$R(t) = \frac{C^{LD}(t)}{C^{DD}(t)} \rightarrow \frac{Z^L}{Z^D} \quad (10)$$

$t_i$  clearly depends on the smearing size and it is determined by observing that  $\Delta E_{DD}$  and  $R$  do not vary appreciably from time to time.

2) For  $t \geq t_i$ , i.e. in the time interval where (9) and (10) have a plateau, we fit the two-point smeared-smeared correlation  $C^{DD}$  to the expression:

$$C^{DD}(t) = (Z^D)^2 e^{-\Delta E_{DD} t} \quad (11)$$

From this fit we obtain the matrix element of the corresponding axial current,  $Z^D$ .

3) We then compute  $R$  defined as:

$$R = \frac{\sum_{t=t_i}^{t_f} R(t) / \sigma_{R(t)}^2}{\sum_{t=t_i}^{t_f} 1 / \sigma_{R(t)}^2} \quad (12)$$

where  $\sigma_{R(t)}$  is the jackknife error on the ratio  $R(t)$  at the time  $t$ . We used  $t_f = 15$  for which the errors in the static-light correlation functions are reasonable.

4)  $Z^L$  is then obtained from the product:

$$Z^L = R \times Z^D \quad (13)$$

where  $Z^D$  is found from the fit of the correlation function to the expression in eq.(11). We will call this method **DD-mass fit**.

We now give two other methods that we have used in this letter to determine  $Z^L$ .

i) **LD-mass fit**: in point 2) we replace the  $C^{DD}$  fit with a fit of  $C^{LD}$ . In this way we obtain the binding energy and  $P = Z^L Z^D$ , i.e. the products of the matrix elements of the local and smeared currents. We then compute  $Z^L$  from the product of  $P$  with  $R$  as defined in 3),  $Z^L = \sqrt{R \times P}$ . The advantage of this method is that  $C^{LD}$  has the smaller statistical errors. A possible disadvantage is that the effective binding energy for  $C^{LD}$  approaches its asymptotic value at large times from below, contrary to the case in which one uses  $C^{DD}$ . However the quality of the plateau that we observe in the effective mass  $\Delta E^{LD}$ , from  $C^{LD}$ , is such that we do not doubt that we have isolated the lightest state. This is shown in

Figure 1:  $\Delta E_{DD}^{eff}$  and  $\Delta E_{LD}^{eff} + 0.2$  from  $C^{DD}$  and  $C^{LD}$  as a function of the time  $t$  for  $L_s = 11$  and  $K = 0.14144$ . We also report the published data of the UKQCD collaboration[7]. The points of ref.[7] have been moved by  $-0.2$  in order to distinguish them from our results. The lines joining the points are there to help the reader distinguish the different cases.

fig.1 where  $\Delta E^{LD}$  and  $\Delta E^{DD}$  are shown together (see also table 2). For comparison on the quality of our data we also report the results of the UKQCD collaboration, at the same value of  $\beta$  and  $K$ , obtained with a statistics of 20 configurations on a  $24^3 \times 48$  lattice. We will compare our static results with those of ref.[7] and our light-light results with ref.[28] at the end of this letter.

ii) **Ratio method:** we compute the following ratio of correlation functions:

$$R_{Z^L}(t_1, t_2) = \frac{C^{LD}(t_1)C^{LD}(t_2)}{C^{DD}(t_1 + t_2)} \quad (14)$$

At large  $t_1$  and  $t_2$  ( $t_{1,2} \geq t_i$ ), it is clear from eqs.(8,11) that  $R_{Z^L} \rightarrow (Z^L)^2$ . In practice, we average  $R_{Z^L}(t_1, t_2)$  from  $t_1 + t_2 = t_m = 2t_i$  to a certain maximum value  $t_1 + t_2 = t_M$ , for which the errors are reasonably small, with all  $t_{1,2}$  which satisfy  $t_{1,2} \geq t_i$ . The advantage of this method is that  $Z^L$  is obtained directly and no fitting is necessary. The possible disadvantage is that we have to compute  $C^{DD}$  at large time distance, i.e.  $\geq 2t_i$ . We note here that this method can be applied not only to measurements of  $f_P^{stat}$ , but to a large variety of matrix elements.

Several consistency checks are possible. One should find a plateau corresponding to the same value of the effective mass for the  $C^{LD}$  and  $C^{DD}$ . Since we extract  $Z^L$  by taking ratios, we also need to overlap the time ranges in which the plateau in the effective mass is observed. We show that this is possible in figs.1,2 where we give  $\Delta E_{DD}^{eff}$  and  $\Delta E_{LD}^{eff}$  as a function of the time for  $L_s = 7 - 13$ .

Figure 2:  $\Delta E_{DD}^{eff}$  and  $\Delta E_{LD}^{eff} + 0.2$  from  $C^{DD}$  and  $C^{LD}$  as a function of the time  $t$  for  $L_s = 7 - 13$  and  $K = 0.14144$ . We also give best estimate,  $\Delta E^{eff} = 0.561(3)$ , the band of which appears as a thick line in the figures. Notice that  $L_s = 11$  works well both for  $C^{LD}$  and  $C^{DD}$  at moderate values of  $t$ .

A more quantitative check of the reliability of the results is given by the fact that different correlation functions give the same effective binding energy. The results and errors for  $\Delta E_{DD}^{eff}$  and  $\Delta E_{LD}^{eff}$  with different smearing sizes and time intervals, which we consider in the good range of  $L_s$  and  $t$ , are reported in table 2. The binding energies extrapolated linearly in  $M_5^2$  to the chiral limit are also given in the same table. From the dependence of the binding energy on the quark mass one can derive the mass difference  $M_{B_s} - M_{B_d}$  [29]. Our best estimate for this quantity is  $M_{B_s}a - M_{B_d}a = 0.020(4)$ , ie.  $M_{B_s} - M_{B_d} = 70(10)$  MeV. We can see from fig.2 and table 2 that the smearing with the best plateau both in the  $DD$  and  $LD$  case is  $L_s = 11$ . Consequently all the results given for  $f_P^{static}$  have been obtained with this smearing size and  $t_i = 5$  and 7.

A further quantitative check is that the three different methods give the same value of  $Z^L$ . A comparison of the different determinations can be done from table 3.  $Z^L$ , extrapolated linearly in  $M_5^2$  to the chiral limit, is also given in the same table. From the dependence of  $Z^L$  on the quark mass we have obtained the value of  $f_{B_s}/f_{B_d} = 1.11(3)$  (see [8]). Finally, we have checked that the  $S$ -smearing (see eq.(5)), when we use  $L_s = 13$ , gives consistent results to the  $D$ -smearing (eq.(6)), thus giving confidence in our methods.

From the results for the effective mass and  $Z^L$  at different values of the Wilson parameter and in the chiral limit, other groups working on the same problem and using the same quark action, but different smearing techniques to isolate the lightest state, can check our results. Our best estimate of  $Z^L$  is 0.111(6) which we obtain from a weighted average of the results in table 3, where the error includes the spread of values in the table added in quadrature with the statistical error.

	$\Delta E_{DD}$			
K	$L_s = 11 \ t = 5-15$	$L_s = 11 \ t = 7-15$	$L_s = 13 \ t = 5-15$	$L_s = 13 \ t = 7-15$
0.14144	0.560(5)	0.556(7)	0.563(6)	0.560(8)
0.14190	0.548(6)	0.545(7)	0.551(7)	0.548(8)
0.14244	0.532(6)	0.529(8)	0.535(8)	0.531(10)
$K_c$	0.514(6)	0.511(9)	0.515(7)	0.513(10)
	$\Delta E_{LD}$			
K	$L_s = 11 \ t = 5-15$	$L_s = 11 \ t = 7-15$	$L_s = 13 \ t = 5-15$	$L_s = 13 \ t = 7-15$
0.14144	0.562(2)	0.561(3)	0.550(2)	0.553(4)
0.14190	0.552(3)	0.551(3)	0.540(3)	0.543(4)
0.14244	0.541(3)	0.539(3)	0.528(3)	0.531(5)
$K_c$	0.526(4)	0.524(5)	—	—

Table 2:  $\Delta E_{DD}$  and  $\Delta E_{LD}$  for  $L_s = 11$  and 13. The fits done in different time intervals have been reported in order to show the stability of the results. The agreement is very good with the only exception of  $\Delta E_{LD}$  when  $L_s = 13$ . The reason can be understood by looking at fig.2 where it is shown that there is a drift in the effective mass as a function of the time. Analogous problems are encountered with  $C^{LD}$  for  $L_s = 7$  and 9. On the contrary this table shows that with  $L_s = 11$  one gets results consistent, within very tiny errors, from both  $C^{LD}$  and  $C^{DD}$  and different time intervals.

Once  $Z^L$  is established for a given value of the  $\beta$ , the determination of  $f_B^{stat}$  still depends upon several factors. Apart from effects of  $O(a)$ , which we expect to be smaller with the SW-Clover action, there are the uncertainties coming from the calibration of the lattice spacing,  $a$ , and from the renormalization constant of the axial current in the static theory (see eq.(1)).

There is only one group which has worked at this value of  $\beta$ , using the SW-Clover action, UKQCD[7, 28]. We now compare our results with theirs for the light meson sector and the static case.

In figs.3 we plot  $M_5^2$  and  $M_V$  as a function of  $1/K$ . Our results are in good agreement for  $M_5^2$  (we find essentially the same  $K_c$ ) but are systematically lower for  $M_V$ . In fact, at the only value of  $K$  which is common to the two calculations, our result coincides with the value of  $M_V$  reported as  $M_V^{low}$  in table 1 of ref.[28]. The results are in better agreement if we include in the comparison the systematic error quoted in ref.[28], which was estimated by looking at the different determinations of  $M_V$ , obtained by varying the time interval of the fits. A possible origin of the difference is that UKQCD fits the correlation functions from  $t_i = 13$  to 23. The difference in  $M_V$  is reflected in the estimates of  $a^{-1}$ , the inverse lattice spacing, which in their case is  $a^{-1} = (2.7 \pm 0.1)$  GeV (no systematic error is reported) to be compared with our value of  $a^{-1} = (3.2 \pm 0.3)$  GeV. On the contrary, our result for  $f_P$  is systematically above the result of ref.[28], see also table 1 and figs. 3. However the results are in this case compatible within the statistical fluctuations. The combined effect of a smaller  $M_V$  and a larger  $f_P$  is that the difference between the two groups becomes a discrepancy for  $f_P/M_V^4$ . From the above discussion we have seen that most of this is due

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<sup>4</sup>Note that we find that  $f_P/M_V/Z_A^{Ren}$  is slightly increasing in the chiral limit, contrary to previous findings at  $\beta = 6.0$ [8]. This is probably due to a statistical effect.

	$Z^L$ from the <b>DD-mass fit</b>	
K	$L_s = 11 \ t = 5-15$	$L_s = 11 \ t = 7-15$
0.14144	0.134(3)	0.132(4)
0.14190	0.127(3)	0.125(4)
0.14244	0.119(3)	0.116(4)
$K_c$	0.108(4)	0.104(5)
	$Z^L$ from the <b>LD-mass fit</b>	
K	$L_s = 11 \ t = 5-15$	$L_s = 11 \ t = 7-15$
0.14144	0.136(2)	0.135(2)
0.14190	0.130(2)	0.129(2)
0.14244	0.123(2)	0.121(2)
$K_c$	0.114(2)	0.111(3)
	$Z^L$ from the <b>Ratio method</b>	
K	$L_s = 11 \ t = 10-15$	$L_s = 11 \ t = 14-15$
0.14144	0.135(2)	0.134(5)
0.14190	0.129(2)	0.128(5)
0.14244	0.121(2)	0.119(6)
$K_c$	0.111(2)	0.109(7)

Table 3:  $Z^L$  obtained from the three methods (see text) using  $L_s = 11$ . The fits are done in two time intervals ( $t = 5 - 15$  and  $t = 7 - 15$ ) in order to show the stability of the results. In the **ratio method** we have chosen  $t_m = 10$  and  $t_M = 15$  for  $t_i = 5$  or  $t_m = 14$  and  $t_M = 15$  for  $t_i = 7$ , see text. The agreement between all three methods is quite satisfactory.

Figure 3:  $M_5^2$ ,  $M_V$ ,  $f_P/Z_A^{Ren}$  and  $f_P/(Z_A^{Ren}M_V)$  as a function of  $1/K$ . We plot both our and the UKQCD[28] results, the latter obtained with a statistics of 60 configurations. Only the statistical errors are shown. The lines are linear fits in  $1/K$  to our data.

to  $M_V$ .

For the static case, at  $K=0.14144$ , our results for the binding energy are definitely incompatible with those of UKQCD[7]. They quote  $\Delta E^{eff} = 0.59 \pm 0.1$  to be compared with our results, given in table 2. At this point a comparison of  $Z^L$  becomes difficult. In any case, the difference for  $Z^L$  is not too large: they find  $Z^L = 0.142_{-6}^{+7}$  at  $K=0.14144$ , to be compared with a typical value (error) for our results of  $Z^L = 0.136(2)$ , and  $Z^L = 0.124_{-7}^{+8}$  at  $K_c$ , to be compared to our estimate of  $Z^L = 0.111(6)$ .

A prerequisite to a precise determination of  $f_B^{stat}$  demands agreement on the value of  $\Delta E^{eff}$  and  $Z^L$  obtained with a given action at a certain value of  $\beta$ . Our group and the UKQCD collaboration have decided to work with the SW-Clover action in order to reduce lattice artefacts. In this paper we have presented a very accurate determination of  $Z^L$ , derived with three different methods, which give compatible results, within tiny statistical errors. Our results for the binding energy show some difference with the results of ref.[7] which, by taking the quoted statistical errors, appear to be of systematic origin. The same can be said for the mass of the vector meson [28]. Other differences, for example  $f_P$ , can instead be the effect of statistical fluctuations.

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